

Effect Of Trade Credit Financing Below The Warehouse Capacity Onstraint In A Bulk Release Pattern

Dr. Ranjeet Singh

Faculty of Commerce and Business Administration
CCS University
Meerut

ABSTRACT :

An inventory model with the combination of two warehouses, deterioration under the effect of trade credits in an inflationary environment has been developed. Own warehouse has limited capacity and rented warehouse has unlimited capacity. Stock in RW is transported to OW in an intermittent pattern (bulk release pattern) which is more practical. The transportation cost for transferring items from RW to OW is taken to be considered. After that, deterioration sets in, and it has been made more realistic and practical by taking Weibull distribution function. Holding costs are taken as variable for both RW and OW and which is linear increasing function of time. This study focuses on an exponentially increasing demand under the conditions of the retailer receiving the supplier trade credit and providing the customer trade credit simultaneously so as to minimize the average total cost. The model incorporates partial backlogging and time horizon is finite. Numerical examples have been presented to explain the theory, while sensitivity of the optimal solution of the system has been studied with respect to various system parameters.

Key words: Warehouse, Bulk, deterioration, environment

INTRODUCTION

Time has affected the business senses of organizations. Every businessman strives to increase his profits, his goodwill, and his customer base. To achieve this, supplier provides a concession or credit limits for a retailer to stimulate the demand, boost market share or decrease inventories of certain items. In framing the traditional inventory model, it was assumed that the payment must be made to the supplier for the items immediately after receiving the consignment. However, in practice, for encouraging the retailer to buy more, the supplier allows a certain fixed period for settling the account and does not charge any interest from the retailer on the amount owed during this period. The traditional economic ordering quantity model considers that the retailer pays the purchasing cost for the product as soon as the products are received; but in reality, the supplier usually offers different delay period, known as trade credit period or deferred payment period, sometimes with different price discounts to encourage the retailer to order more quantity.

Inflation plays a very interesting and significant role: it increases the cost of goods. To safeguard from the rising prices, during the inflation regime, the organization prefers to keep a higher inventory, thereby increasing the aggregate demand. The effect of time value of money is very important and should be reflected in the development of inventory models.

Formulation and Solution of the Model

It is assumed that the management owns a warehouse with a fixed capacity of W units and any quantity exceeding this should be stored in a rented warehouse, is assumed to be available with abundant space. i.e., we assume that a company purchases Q ($Q > W$) units out of which W units are kept in OW and $(Q - W) = I_r$ units are kept in RW. Initially, the demands are not using the stocks of OW until the stock level drops to $(W - K)$ units at the end of T_1 then a lot is released from the RW to replenish the stock level back to W in OW. Hence at time T_1 , first lot is released from RW. Up to this time the stocks in RW were being depleted due to decay only. As a result, the stock level of OW again becomes W and the stocks of OW are used to meet further demands. The stock level in OW depletes due to both demand and deterioration. This process is continued until the stock in RW is fully exhausted. After the last shipment, only W units are used to satisfy

the demand during the time interval $[T_{n-1}, T_n]$ and then the shortages occur and partially backlogged during the time interval $[T_n, T]$. The graphical representation of the whole process is shown in the figures 1.1 and 1.2. The inventory level at RW and OW are governed by the following differential equations:

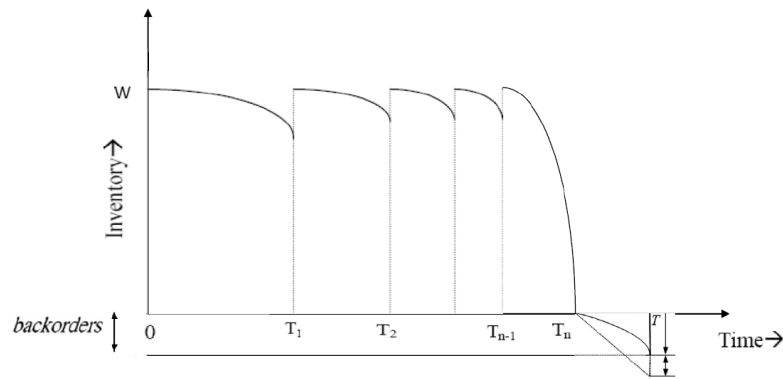


Fig 1.1 : Graphical representation of inventory system for own warehouse

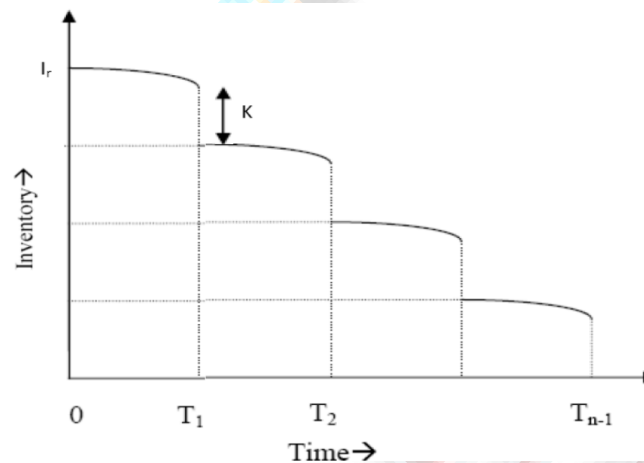


Fig 1.2 : Graphical representation of inventory system for rented warehouse

For Own warehouse

$$\frac{dI_0(t)}{dt} + \alpha\beta t^{\beta-1} I_0(t) = -ae^{bt}, \quad T_i \leq t \leq T_{i+1} \quad \dots (1.1)$$

With the boundary condition $I_0(T_i) = W$, $I = 0, 1, 2, \dots, n-1$ and $i = n$, $I_0(T_i) = 0$ also $T_0 = 0$

For Rented Warehouse

$$\frac{dI_r(t)}{dt} + ght^{h-1} I_r(t) = 0, \quad T_i \leq t \leq T_{i+1} \quad \dots (1.2)$$

With the boundary condition $I_r(0) = I_r$, for $0 \leq t \leq T_1$ and $I_r(T_{i+1}) = I_r(T_i) - K$ for $T_i \leq t \leq T_{i+1}$, $i = 1, 2, \dots, n-2$.

Using the boundary conditions, we arrive at the following results for the own warehouse

$$I_0(t) = ae^{-\alpha t^\beta} \left[(T_i - t) + \frac{b}{2} (T_i^2 - t^2) + \frac{\alpha}{\beta+1} (T_i^{\beta+1} - t^{\beta+1}) + \frac{b^2}{6} (T_i^3 - t^3) + \frac{\alpha^2}{2(2\beta+1)} (T_i^{2\beta+1} - t^{2\beta+1}) \right] + We^{\alpha(T_i^\beta - t^\beta)} \quad \dots (1.3)$$

$$I_0(t) = ae^{-\alpha t^\beta} \left[(T_n - t) + \frac{b}{2} (T_n^2 - t^2) + \frac{\alpha}{\beta+1} (T_n^{\beta+1} - t^{\beta+1}) + \frac{b^2}{6} (T_n^3 - t^3) + \frac{\alpha^2}{2(2\beta+1)} (T_n^{2\beta+1} - t^{2\beta+1}) \right] \quad \dots (1.4)$$

$$I_r(t) = I_r e^{-g^h t}, \quad 0 \leq t \leq T_1 \quad \dots (1.5)$$

$$I_r(t) = [I_r(T_i) - K] e^{g(T_{i+1} - t)}, \quad T_i \leq t \leq T_{i+1}, i = 1, 2, \dots, n-2 \quad \dots (1.6)$$

During the time interval $[T_n, T]$

$$\frac{dI_s(t)}{dt} = -\frac{ae^{bt}}{1 + \delta(T-t)}, \quad T_n \leq t \leq T \quad \dots (1.7)$$

With the boundary condition $I_0(T_n) = 0$, solution of the equation (1.7)

$$I_0(t) = a \left[(1 - \delta T)(T_n - t) + \frac{1}{2}(b - b\delta T + \delta)(T_n^2 - t^2) + \frac{b\delta}{3}(T_n^3 - t^3) \right] \quad \dots (1.8)$$

These equations clearly show the variation of inventory in the prescribed time period in both the OW and the RW.

Present worth Set-up Cost

Order is placed at the beginning of each cycle and hence for every cycle,

$$SPC = C_0 \quad \dots (1.9)$$

Present worth Item Cost

Inventory is bought at the beginning of the cycle and stored separately at the two warehouses. Hence

$$IC = c(I_r + W) \quad \dots (1.10)$$

Present worth Holding Cost in OW

Inventory is available during $T_i \leq t \leq T_{i+1}$, $i = 0, 1, 2, \dots, n-1$.

Hence the holding cost needs to be computed during these time periods

$$\begin{aligned} HC_{OW} &= \sum_{i=0}^{n-1} e^{-rT_i} \int_{T_i}^{T_{i+1}} I_0(t)(C_{h1} + \phi t)e^{-rt} dt \\ &= \sum_{i=0}^{n-2} e^{-rT_i} \int_{T_i}^{T_{i+1}} (C_{h1} + \phi t)I_0(t)e^{-rt} dt + e^{-rT_{n-1}} \int_{T_{n-1}}^{T_n} (C_{h1} + \phi t)I_0(t)e^{-rt} dt \end{aligned} \quad \dots (1.11)$$

Present worth holding cost in RW

Inventory is available during $0 \leq t \leq T_1$ and $T_i \leq t \leq T_{i+1}$, $i = 1, 2, \dots, n-2$.

Hence the holding cost needs to be computed during these time periods

$$\begin{aligned} HC_{OW} &= \sum_{i=0}^{n-2} e^{-rT_i} \int_{T_i}^{T_{i+1}} I_r(t)(C_{h2} + \phi t)e^{-rt} dt \\ &= \int_0^{T_1} I_r(t)(C_{h2} + \phi t)e^{-rt} dt + \sum_{i=1}^{n-2} e^{-rT_i} \int_{T_i}^{T_{i+1}} I_r(t)(C_{h2} + \phi t)e^{-rt} dt \end{aligned} \quad \dots (1.12)$$

Present worth Shortage cost or backorder cost

Shortages occur during $T_n \leq t \leq T$, Hence the shortage cost needs to be computed during this time period

$$SC = C_b e^{-rT_n} \int_{T_n}^T [-I_s(t)] e^{-rt} dt \quad \dots (1.13)$$

Present worth Opportunity Cost

Opportunity cost needs to be computed during the time period T_n to T .

$$OC = C_1 e^{-rT_n} \int_{T_n}^T a e^{bt} \left[1 - \frac{1}{1 + \delta(T-t)} \right] e^{-rt} dt \quad \dots (1.14)$$

Present worth Transportation Cost

$$TRC = T_C \sum_{i=1}^{n-1} e^{-rT_i} \quad \dots (1.15)$$

Interest payable

When the end point of credit period is shorter then or equal to the length of period with positive inventory stock o the item ($M \leq T_n$), payment for goods is settled and the retailer starts paying he capital opportunity cost for the items in stock with rate I_p . Thus, the opportunity cost per cycle (Interest payable) is given below.

Case 1.1: $M > T_n$

$$IP_1 = cI_c e^{-rM} \int_M^{T_n} I_0(t) e^{-rt} dt \quad \dots (1.16)$$

Case 1.2: $M > T_n$

In this case there is no opportunity cost.

$$\text{Therefore} \quad IP_2 = 0 \quad \dots (1.17)$$

Interest Earned from sales Revenue

There are many different ways to tackle the interest earned. Here we assume that during the time when the account is not settled, the retailer sells the goods and continues to accumulate sales revenue and earns the interest with rate I_e .

Therefore interest earned per cycle for two different cases is given below

Case 1.1: $M > T_n$

$$IE_1 = pI_e \left\{ \sum_{i=0}^{n-1} e^{-rT_i} \int_{T_i}^{T_{i+1}} (ae^{bt}) te^{-rt} dt \right\} \quad \dots (1.18)$$

Case 1. 2: $M > T_n$

$$IE_2 = pI_e \left[\sum_{i=0}^{n-1} e^{-rT_i} \int_{T_i}^{T_{i+1}} (ae^{bt}) te^{-rt} dt + (M - T_n) \left\{ \sum_{i=0}^{n-1} e^{-rT_i} \int_{T_i}^{T_{i+1}} (ae^{bt}) e^{-rt} dt \right\} \right] \quad \dots (1.19)$$

Present worth Total Profit

The net present worth total cost is formed by deducing various costs, we get

$$Z = \text{Setup Cost} + \text{Holding Cost in RW} + \text{Holding Cost in OW} + \text{Shortage Cost} + \text{Opportunity Cost} + \text{Interest Payable} - \text{Interest Earned} \quad \dots (1.20)$$

To acquire the optimal values of time that minimizes the total cost with the help of software Mapple

NUMERICAL ILLUSTRATION

The model developed above is illustrated by the following numerical example. Numerical data is based on the previous study in standard units.

Example: For Case 1.1. ($T_{n-1} \leq M \leq T_n$) and Case 2. ($T_n \leq M \leq T$)

$$\begin{array}{lllll} \alpha = 0.05, & \beta = 2, & g = 0.02, & h = 2, & r = 0.06, \\ W = 500, & C_0 = 100, & p = 10, & T_C = 50, & C_1 = 10, \\ C_{h1} = 5, & C_{h2} = 7, & C_b = 8, & I_c = 0.12, & I_e = 0.15, \\ M = 1.5, & \delta = 0.5, & a = 10, & b = 0.2, & T = 10, \\ T_0 = 0, I_r = 2500, & K = 200, & \phi = 0.004 & \phi = 0.05 & \end{array}$$

Table 1.1 : For Case 1 $T_{n-1} \leq M \leq T_n$ Optimal Values of Time and Total cost for different number of cycles

N	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	Total Cost (TC)
3	2.9526	5.9052	7.2211	-	-	-	-	21225.8
4	1.9932	1.9864	5.9796	7.2341	-	-	-	23529.5
5	1.5500	1.1000	4.6500	6.2000	7.2889	-	-	27091.3
6	1.1836	2.3672	1.5508	4.7344	5.9180	7.2239	-	23262.0
7	0.9887	1.9774	2.9661	1.9548	4.9435	5.9322	7.2255	23478.0

Here, we observe that as the number of cycles increases, the total cost increases from cycle 3 to 5, but the total cost per unit time is found to be minimum for $n = 3$ and it is observed that the total cost for number of cycle 6,7 is greater than that of number of cycle of 1. Optimal values of time are decrease for increasing the number of cycles respectively.

Table 1.2 : For Case1.2 $T_n \leq M \leq T$ Optimal Values of Time and Total cost for different number of cycles

N	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	Total Cost (TC)
3	2.9527	5.9055	6.4965	-	-	-	-	21181.8
4	1.7135	1.4270	5.1405	6.7451	-	-	-	24010.7
5	1.5503	1.1006	4.6509	6.2012	6.9987	-	-	27027.7
6	1.1847	2.3694	1.5541	4.7388	5.9238	6.1082	-	23474.0
7	0.9887	1.9774	2.9661	1.9548	4.9435	5.9322	6.9892	23377.7

Here, it is also observe that the optimal number of cycle is 1 as we observe that as the number of cycles increases, the total cost increases from cycle 3 to 5, but the total cost per unit time is found to be minimum for $n = 3$ and it is observed that the total cost for number of cycle 6,7 is greater than that of number of cycle of 1. Optimal values of time are decrease for increasing the number of cycles respectively.

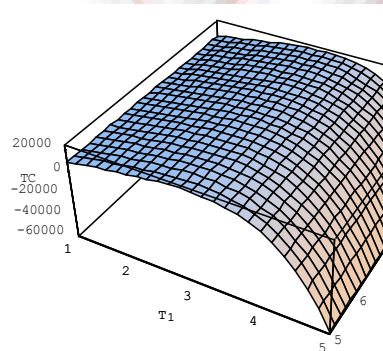


Fig 1.3 : Convexity of the Total cost of Case 1. ($T_{n-1} \leq M \leq T_n$) and Case 2. ($T_n \leq M \leq T$) on similar pattern for $n = 3, 4$

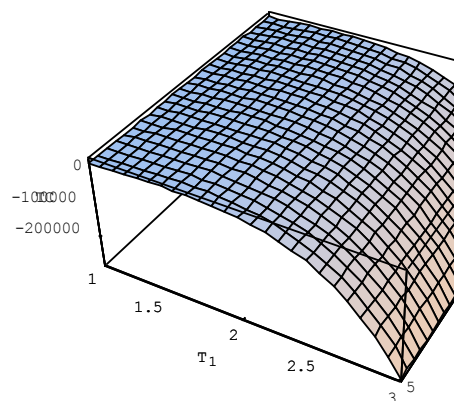


Fig 1.4 : Convexity of the Total cost of Case 1. ($T_{n-1} \leq M \leq T_n$) and Case 2. ($T_n \leq M \leq T$) on similar pattern for $n = 5, 6, 7$

The values of the total cost of Case 2. ($T_n \leq M \leq T$) are comparatively less than the Case 1. ($T_{n-1} \leq M \leq T_n$) The optimal number of cycles here also is one.

CONCLUSION

The impact of bulk release pattern on an order-level inventory model with two levels of storage for an item subject to Weibull distribution decays in inflationary environment has been considered. Trade credit policy plays an important role in the business of many products and it serves the interests of both the supplier and retailer. The supplier usually expects the profit to increase since rising sales volumes compensate the capital losses incurred during the credit period. Also, the supplier finds an effective means of price discrimination which circumvents anti-trust measures. On the contrary, the retailer earns an interest by investing the sale proceeds earned during the credit period. Therefore trade credit policy is provided to the customers to attract them and boost up the demand. A rented warehouse is used when the ordering quantity exceeds the limited capacity of the own warehouse.

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